Student Workbook

Simplifying and Proving Trigonometric Identities

Name: ________________________________ Date: ________________ Class: _________

ESSENTIAL QUESTION

How can you use known trigonometric relationships to rewrite an expression in a simpler or more useful form?

Lesson Overview

A trigonometric identity is an equation that is true for all values of the variable for which both sides are defined — unlike a conditional equation, which may hold only for specific values. The three families of fundamental identities are Pythagorean, reciprocal, and quotient. Strategy for proving identities: work on one side only (usually the more complex side). Rewrite using known identities, factor, combine fractions, or multiply by a conjugate until the two sides match. Never move terms across the equals sign.

$$\sin^2\theta + \cos^2\theta = 1$$
$$1 + \tan^2\theta = \sec^2\theta$$
$$1 + \cot^2\theta = \csc^2\theta$$

Recip.	$\csc\theta = 1/\sin\theta$; $\sec\theta = 1/\cos\theta$; $\cot\theta = 1/\tan\theta$
Quotient	$\tan\theta = \sin\theta/\cos\theta$; $\cot\theta = \cos\theta/\sin\theta$
Strategy	Work on ONE side only. Never move terms across the equals sign.

Worked Examples

EXAMPLE 1

Simplify: $\sin^2\theta \cdot \sec^2\theta + \sin^2\theta$

- Use $\sec^2\theta = 1/\cos^2\theta$: $\sin^2\theta/\cos^2\theta + \sin^2\theta$
- Recognize $\sin^2\theta/\cos^2\theta = \tan^2\theta$

Answer: $\tan^2\theta + \sin^2\theta$

EXAMPLE 2

Simplify: $(1 - \cos^2\theta) / \sin\theta$

- Recognize $1 - \cos^2\theta = \sin^2\theta$ (Pythagorean identity)
- Substitute: $\sin^2\theta / \sin\theta$
- Cancel one factor of $\sin\theta$ (assuming $\sin\theta \neq 0$)

Answer: $\sin\theta$

EXAMPLE 3

Prove the identity: $\tan\theta \cdot \cos\theta = \sin\theta$

- Start with the left side: $\tan\theta \cdot \cos\theta$
- Replace $\tan\theta = \sin\theta/\cos\theta$ (quotient identity): $(\sin\theta/\cos\theta) \cdot \cos\theta$
- Cancel $\cos\theta$: $\sin\theta$. Left side equals right side ✓

Answer: Identity proved: $\tan\theta \cdot \cos\theta = \sin\theta$

EXAMPLE 4

Prove: $\sec\theta - \cos\theta = \sin\theta \cdot \tan\theta$

- Work on the left side: $\sec\theta - \cos\theta = 1/\cos\theta - \cos\theta$
- Combine over a common denominator: $(1 - \cos^2\theta)/\cos\theta = \sin^2\theta/\cos\theta$
- Split: $\sin\theta \cdot (\sin\theta/\cos\theta) = \sin\theta \cdot \tan\theta$ ✓

Answer: Identity proved: $\sec\theta - \cos\theta = \sin\theta \cdot \tan\theta$

EXAMPLE 5

Simplify: $(\sec^2\theta - 1) / \sec^2\theta$

- Use $\sec^2\theta - 1 = \tan^2\theta$ (Pythagorean identity): $\tan^2\theta / \sec^2\theta$
- Replace $\tan^2\theta = \sin^2\theta/\cos^2\theta$ and $\sec^2\theta = 1/\cos^2\theta$: $(\sin^2\theta/\cos^2\theta) \cdot \cos^2\theta$
- Cancel $\cos^2\theta$

Answer: $\sin^2\theta$

Guided Practice

Work through each problem below. Use the hint if you get stuck — write your final answer on the last line.

1 Simplify: $\cos^2\theta \cdot \csc^2\theta - \cos^2\theta$

Hint: Factor out $\cos^2\theta$, then use a Pythagorean identity on what remains inside the parentheses.

2 Simplify: $\sin\theta \cdot \cot\theta$

Hint: Replace $\cot\theta$ with $\cos\theta/\sin\theta$ and cancel.

3 Prove: $\csc\theta \cdot \cos\theta = \cot\theta$

Hint: Replace $\csc\theta$ with $1/\sin\theta$ on the left side.

4 Prove: $(1 + \tan\theta)^2 = \sec^2\theta + 2\tan\theta$

Hint: Expand the left side, then use $1 + \tan^2\theta = \sec^2\theta$.

5 Simplify: $(\csc^2\theta - 1) / \csc^2\theta$

Hint: Use $\csc^2\theta - 1 = \cot^2\theta$, then rewrite $\cot^2\theta/\csc^2\theta$ in terms of sin and cos.

Key Vocabulary

Trigonometric Identity	An equation involving trig functions that is true for ALL values where both sides are defined.
Conditional Equation	An equation true only for specific values of the variable, not universally (unlike an identity).
Pythagorean Identity	$\sin^2\theta+\cos^2\theta=1$ and its two variants, derived by dividing through by $\cos^2\theta$ or $\sin^2\theta$.
Reciprocal Identity	$\csc\theta=1/\sin\theta$, $\sec\theta=1/\cos\theta$, $\cot\theta=1/\tan\theta$ — each function paired with its reciprocal.
Quotient Identity	$\tan\theta=\sin\theta/\cos\theta$ and $\cot\theta=\cos\theta/\sin\theta$.
Simplify	Rewrite a trig expression in an equivalent but shorter or more useful form.
Prove (an identity)	Show that one side of an equation can be transformed, step by step, into the other side.
Conjugate	A paired expression (like $1+\sin\theta$ for $1-\sin\theta$) used to clear a difference-of-squares form from a denominator.

Check Your Understanding

Circle the letter of the correct answer for each question.

1 **Which identity states that $\sin^2\theta + \cos^2\theta = 1$?** Multiple Choice

A Quotient identity
B Reciprocal identity
C Pythagorean identity
D Even/odd identity

2 **Simplify: $\sin\theta/\cos\theta$** Multiple Choice

A $\sec\theta$
B $\csc\theta$
C $\cot\theta$
D $\tan\theta$

3 **Which expression is equivalent to $1 - \sin^2\theta$?** Multiple Choice

- **A** $\tan^2\theta$
- **B** $\cos^2\theta$
- **C** $\sec^2\theta$
- **D** $\csc^2\theta$

4 **Simplify: $\cos\theta \cdot \tan\theta$** Multiple Choice

- **A** 1
- **B** $\sin\theta$
- **C** $\cos\theta$
- **D** $\sec\theta$

5 **To prove an identity, you should:** Multiple Choice

- **A** Move terms from one side to the other
- **B** Work on both sides simultaneously
- **C** Transform one side until it matches the other
- **D** Substitute a specific angle value

Common Mistakes to Avoid

✗ Moving terms across the equals sign when proving an identity (e.g., adding $\cos\theta$ to both sides).

✓ Work on one side only. Transform it using identities until it equals the other side.

✗ Confusing $\sec^2\theta - 1 = \tan^2\theta$ with $\sec^2\theta + 1 = \tan^2\theta$.

✓ The correct form is $\sec^2\theta - 1 = \tan^2\theta$ (subtract 1 from $\sec^2\theta = 1 + \tan^2\theta$).

✗ Writing $1/\sin^2\theta = \cos^2\theta$.

✓ $1/\sin^2\theta = \csc^2\theta$. The reciprocal of $\sin^2\theta$ is $\csc^2\theta$, not $\cos^2\theta$.

✗ Cancelling $\sin\theta$ from $\sin\theta + \cos^2\theta/\sin\theta$ as if $\sin\theta$ were a factor of the whole expression.

✓ Only cancel common factors. Here $\sin\theta$ is a term in the numerator, not a factor of the entire expression.

Math Tips

★ When stuck proving an identity, try converting everything to $\sin\theta$ and $\cos\theta$ — it almost always reveals the path.

★ The three Pythagorean identities all come from $\sin^2\theta+\cos^2\theta=1$. Divide by $\cos^2\theta$ or $\sin^2\theta$ to get the other two.

★ If you see a difference of squares like $1-\cos^2\theta$ or $\sec^2\theta-1$, immediately replace it with the Pythagorean equivalent.

★ Multiplying numerator and denominator by a conjugate (e.g., $1+\sin\theta$) is powerful when you see $1-\sin\theta$ in a denominator.

★ Always check the domain: identities hold wherever both sides are defined; $\tan\theta$ is undefined at $\theta=\pi/2+n\pi$.